A COMPARISON OF ECONOMETRIC MARKET
SIMULATIONS OF PARETO OPTIMA WITH
MATHEMATICAL PROGRAMMING CALCULATIONS
OF PARETO OPTIMA

WHITE PAPER

A Comparison of Econometric Market Simulations of Pareto Optima with Mathematical Programming Calculations of Pareto Optima

by

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INTRODUCTION

The purpose of this paper is to discuss two modeling approaches for RPA analysis. It is taken as a given that the objective of both approaches is to determine Pareto optimum levels of output of forest and rangeland resources. In this context, the appropriate land base is all of the nation's forest and rangeland, but land in public ownership is emphasized because it is in direct public sector control.

The two alternative approaches are:

1.) Market equilibrium simulations with econometrically derived supply and demand curves.

2.) Mathematical progamming (operations research models) that optimizes an objective function constructed out of demand curves and factor costs (factor prices), subject to production constraints.

Both of these approaches are theoretically consistent with the conditions for Pareto optima. And, both approaches utilize demand curves as marginal benefit functions. The basic difference is that the econometric approach assumes that the regressed supply curve reflects cost minimization at all levels of output. The operations research approach, on the other hand, determines a cost efficient solution with only factor costs given. In other words, the operations research models determine the cost minimizing factor mix for the level of output in solution, while the econometric approach assumes that the observed cost/output data reflect cost minimizing behavior.

Another difference between the two approaches occurs because of pragmatic considerations. Specifically, the econometric approach must generally be carried out in a functional (single resource) manner. The operations research approach, on the other hand is quite conducive to integrated (multi-resource) analysis. Both of these differences are discussed further in the next two sections.

COST EFFICIENCY

Let us begin this discussion with a brief review of economic theory involving cost functions and supply curves. For simplicity, assume two inputs and one output with a production function:

$$Y = f(X_1, X_2)$$

where the Y is output and the X's are inputs.

This implies supply and demand functions as:

$$D \quad P_y = D\ (Y, B_j);\ j = 1, n$$

$$S \quad P_y = S\ (Y, A_i);\ i = 1, m$$

Supply curves are derived from the profit maximizing condition of value marginal product pricing of all factors:

$$P_y \frac{\partial f}{\partial X_1} = P_{x^1},$$

$$P_y \frac{\partial f}{\partial X_2} = P_{x^2}, \qquad (1)$$

and marginal cost pricing of outputs:

$$P_y = MC^y{}_{x_1,\ x_2}.$$

In other words, each point on the supply curve indicates the minimum marginal cost of producing the given level of Y. This minimum marginal cost is obtained by utilizing conditions (1) in determining the input mix (X_1, X_2). It is important to note that the supply curve reflects a priori cost minimizing combinations of X_1 and X_2 for each level of Y.

Now, the econometric approach regresses estimates of the supply and demand functions on observed data. If the data sets used reflect profit-maximizing behavior, then conditions (1) would be expected to hold and the estimated supply curve would reflect cost efficiency. The resulting

calculated equilibrium, everything else being correct, can be regarded as Pareto optimal. This approach is thus tenable (in terms of cost efficiency) when data sets are available that "contain" profit maximizing behavior. In those cases where the relevant commodities are preponderately provided through the public sector, such data sets may not be available, and econometric derivation of supply curves would be impossible.

The operations research approach attempts to:

Maximize $H = \int_0^{P_y} D(Y, \bar{B}_j)\, dP_y - (P_{x_1}X_1 + P_{x_1}X_2)$

Subject to: $Y = f(X_1, X_2)$

Thus, if we define:

$$R(Y) = \int_0^{P_y} D(Y, \bar{B}_j)\, dP_y$$

Then, substituting $f(X_1, X_2)$ for Y:

$$H = R[f(X_1, X_2)] - P_{x_1}X_1 - P_{x_2}X_2$$

First order conditions for maximizing H are derived as:

$$\frac{\partial H}{\partial X_1} = \frac{\partial R}{\partial f}\frac{\partial f}{\partial X_1} - P_{x_1} \overset{\text{Set}}{=} 0$$

$$\frac{\partial H}{\partial X_2} = \frac{\partial R}{\partial f}\frac{\partial f}{\partial X_2} - P_{x_2} \overset{\text{Set}}{=} 0$$

by the chain rule.

This is equivalent to:

$$D(Y, \bar{B}j) \cdot \frac{\partial f}{\partial X_1} = P_{X_1}$$

$$D(Y, \bar{B}_j) \cdot \frac{\partial f}{\partial X_2} = P_{X_2}$$

Since $P_y = D(Y, \bar{B}_j)$, this is the equivalent to (1) for the case of nonfixed P_y.

This demonstrates that the operations research approach determines the cost minimizing combinations of x_1 and x_2 rather than assumming them to exist in observed data. This is <u>not</u> to say that one method is "better" than the other. Rather, it points up the fact that each method is designed to solve a <u>different problem</u>. The econometric approach is designed to identify behavioral functions that economic theory indicates to be appropriate. The operations research approach is designed to solve for the efficiency conditions that, according to economic theory, creates (part) of that behavior. The point is made graphic if one imagines a set of firms utilizing operations research models in determining their output and cost levels, followed by econometric regression by an outside analyst on those output and cost figures to derive a predictive supply function.

It is also important to note that the operations research analysis utilizes the demand functions (for output benefits) that the econometric analysis derived. Demand functions are regressed based on utility maximizing assumptions on observed data similar to the cost efficiency assumptions for supply curves. The distinction is that the utility maximizing assumptions concern behavior of consumers, and are not affected by public ownership of the production unit. Note: the observed equilibriums may be distorted by public ownership where equilibrium prices are not charged, but the assumptions of utility maximization concerning the demand curve itself are left intact. The task of deriving demand curves for non-market goods that are associated with no cost-efficient supply data is discussed in the "Recommendations" section.

SINGLE vs. MULTI-RESOURCE EQUILIBRIA

Another difference between the two approaches occurs simply due to pragmatic reasons. Specifically, it is essentially impossible at this time to perform econometric derivation of supply and demand functions for more than one type of renewable resource (e.g. timber, recreation, fish and wildlife, water, forage) at a time. Econometric analysis thus tends to be functional. Though some operations research efforts have been functional (e.g. TimberRAM, Model II, MUSYC), other efforts have involved multiple resource management and planning (e.g. FORPLAN, NIMRUM, GREATIAM, MAGE5).

Ignoring the previous discussion concerning the basic differences in the type of problem each approach is oriented towards, it will be useful to discuss functional as opposed to multiple resource supply analysis. It will suffice to define a situation with two inputs and two outputs:

$$0 = f(X_1, X_2, Y_1, Y_2)$$

where f is an implicit production function, the X's are inputs and the Y's are outputs. Assuming linear forms for convenience, this joint production system implies supply functions of the form:

$$S_1: \quad P_1 = a_1 + b_{11}Y_1 + b_{12}Y_2$$

$$S_2: \quad P_2 = a_2 + b_{21}Y_1 + b_{22}Y_2$$

where the P's are prices of the Y's. Supply shifters are suppressed for simplicity. The cross-supply terms ($b_{21}Y_1$ and $b_{12}Y_2$) appear because of the joint production function assummed.

Integrated analysis would attempt to find the equilibrium point between these supply functions and the demand functions for Y_1 and Y_2. These demand functions will be assumed independent (no cross terms) for this analysis. This assumption is tenable if the two outputs are neither substitutes nor compliments in consumption (e.g. plywood and recreation experiences).

Thus, integrated analysis would attempt to solve the system of equations:

$$D_1:\ P_1 = c_1 + d_{11}Y_1$$

$$D_2:\ P_2 = c_2 + d_{22}Y_2$$

(2)

$$S_1:\ P_1 = a_1 + b_{11}Y_1 + b_{12}Y_2$$

$$S_2:\ P_2 = a_2 + b_{21}Y_1 + b_{22}Y_2$$

The functional approach necessarily assumes that the production of Y_1 and Y_2 are independent. One example of such a production situation is:*

$$Y_1 = f_1(X_1)$$

$$Y_2 = f_2(X_2).$$

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*It is important to note that:

$$Y_1 = f_1(X_1,\ X_2)$$

$$Y_2 = f_2(X_1,\ X_2)$$

is a joint production system that will result in non-zero cross supply terms.

This sort of independent production structure implies supply functions in the form (demand functions are also included):

$$S_1:\ P_1 = a_1 + b_{11}Y_1$$

$$S_2:\ P_2 = a_2 + \qquad b_{22}Y_2$$

(3)

$$D_1:\ P_1 = c_1 + d_{11}Y_1$$

$$D_2:\ P_2 = c_2 + \qquad d_{22}Y_2$$

Given that, at the very least, public budgets (capital) and land inputs are potentially employed simultaneously in producting all renewable resource outputs, the functional production structure (separable output production) would seem to be inappropriate. The question, though, is really: "will the equilibrium quantity solution of (2) be any different than that of (3)?"

The equilibrium solution to (3) is:

$$Y_1 = \frac{a_1 - c_1}{d_{11} - b_{11}}$$

$$Y_2 = \frac{a_2 - c_2}{d_{22} - b_{22}}$$

by setting supply and demand prices equal. Similarly, the equilibrium solution to (2) is:

$$Y_1 = \frac{\left(\frac{b_{12}}{d_{11} - b_{11}}\right)\left(\frac{a_2 - c_2}{d_{22} - b_{22}}\right) + \frac{a_1 - c_1}{d_{11} - b_{11}}}{1 - \left(\frac{b_{12}}{d_{11} - b_{11}}\right)\left(\frac{b_{21}}{d_{22} - b_{22}}\right)}$$

$$Y_2 = \frac{\left(\frac{b_{21}}{d_{22} - b_{22}}\right)\left(\frac{a_1 - c_1}{d_{11} - b_{11}}\right) + \frac{a_2 - c_2}{d_{22} - b_{22}}}{1 - \left(\frac{b_{12}}{d_{11} - b_{11}}\right)\left(\frac{b_{21}}{d_{22} - b_{22}}\right)}$$

This demonstrates that the equilibrium levels of Y_1 and Y_2 can only be expected to be the same if $b_{12} = b_{21} = 0$. This is not surprising since these are the coefficients that distinguish the integrated from the functional approach.

Another interesting question is: "will the benefit and cost (net benefit) information derived from (2) be similar to that from (3)?" In order to answer this question rigorously, construct the net benefit functions based on (2) and (3).

The integrated net benefit function is:

$$\sum_{i=1}^{2} {}_{o}\int^{Y_i} (c_i + d_{ii}Y_i)dY_i - {}_{c}\int \sum_{i=1}^{2}(a_i + \sum_{j=1}^{2} b_{ij}Y_i)dY_i$$

where ${}_{c}\int$ is the line integral of the system of supply curves between the origin and the (Y) terminal output vector.

The functional net benefit function is:

$$\sum_{i=1}^{2} {}_{o}\int^{Y_i} (c_i + d_{ii}Y_i)dY_i - \sum_{i=1}^{2} {}_{o}\int^{Y_i} (a_i + b_{ii}Y_i)dY_i$$

Clearly, these two net benefit functions differ only on the cost side and will be equivalent only <u>if</u>:

$${}_{c}\int \sum_{i=1}^{2} (a_i + \sum_{j=1}^{2} b_{ij}Y_i)dY_i \overset{?}{=} \sum_{i=1}^{2} {}_{o}\int^{Y_i} (a_i + b_{ii}Y_i)dY_i$$

By a common theorem on line integrals (Taylor, 1955, p. 437):

$$ {}_{c}\int \sum_{i-1}^{2} (a_i + \sum_{j=1}^{2} b_{ij} Y_i) dY_i = \underset{\sim}{a}'\underset{\sim}{Y} + \tfrac{1}{2}\underset{\sim}{Y}'\underset{\sim}{b}\underset{\sim}{Y} $$

$$ = \sum_{i=1}^{2} a_i Y_i + \tfrac{1}{2} \sum_{i=1}^{2} Y_i \sum_{j=1}^{2} b_{ij} Y_j, \qquad (4) $$

so long as $\sum_{i=1}^{2} a_i + \sum_{j=1}^{2} b_{ij} Y_i$ is single-valued over the range $0\text{-}\underset{\sim}{Y}$.

By simple integration:

$$ \sum_{i=1}^{2} {}_{o}\int^{Y_i} (a_i + b_{ii} Y_i) dY_i = \sum_{i=1}^{2} a_i Y_i + \tfrac{1}{2} b_{ii} Y_i^2 \qquad (5) $$

And (4) differs from (5) by the terms:

$$ \tfrac{1}{2} b_{12} Y_1 Y_2 + \tfrac{1}{2} b_{21} Y_1 Y_2 = (4) - (5). $$

Economic theory indicates that cross supply terms are symmetrical*, due to the absense of an income effect (Henderson and Quandt, 1971, p.98). Thus, the difference between (4) and (5) is:

$$ b_{12} Y_1 Y_2 \text{ or } b_{21} Y_1 Y_2. $$

It is clear that with non-trivial cross-supply coefficients (b_{12} and b_{21}), the potential error in (5) is substantial.

The sign of the b_{12} and b_{21} is determined by the nature of the joint production function involved. If they are negative, functional supply curves over-estimate joint costs. If they are positive, functional supply curves under-estimate joint costs. In a system of n goods, some cross supply terms might be positive and some negative, in which case the overall bias would be difficult to predict.

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*This condition is necessary for the line integral to be independent of the path of integration (c) and thus single-valued.

This suggests that functional derivation of supply and demand functions will not result in theoretically tenable results if, in fact, the relevant production system is "joint" and the resulting supply functions have non-zero cross supply terms. The biases derived above suggest that if cross-supply coefficients are close to zero, then the errors involved in functional equilibriums will be small.

Before proceding, it is important to note that the operations research approach does not derive the system of supply functions described in (2), _per se_. Rather, single output supply function are never considered. Costs are included as costs of "management prescriptions" which are joint costs associated with an entire vector of outputs. This joint cost is equivalent to (4), because it is the total cost of the output vector with all production interactions already having been accounted for. The implicit system of supply functions may not, in this case, be linear, however. The use of joint costs does not allow derivation of cost/benefit ratios for each output, but this is actually impossible anyway. Refering to (2), a change in (for example) Y_1 would cause not only a shift along its own supply function, but a shift of Y_2's supply function and thus re-equilibration of all prices and quantities. This simply reflects the fact that allocation of a joint cost to any individual output is purely arbitrary.

RECOMMENDATION

It must be emphasized that nothing said here should be taken to reduce the importance of econometric demand estimation. And, in the case of positively sloped supply functions, demand and supply must be indentified simultaneously (through some method such as two-stage least squares). The basic recommendation of this paper is to utilize econometrically derived demand functions as marginal benefit functions in the operations research models together with joint costs (costs based on management prescriptions and associated with entire output vectors) to determine Pareto optimal solutions for public resource allocation problems.

One glaring question remains: How can the demand curves (which pragmatically must be derived functionally) be estimted if functional supply curves are untenable? Recommendations are as follows:

1.) For commodities where market data are available, only a single output is generally involved, and cost efficient behavior is a tenable assumption. This would include privately produced timber, forage and commercial uses of water and fish and wildlife. Thus, econometric regression of functional supply and demand curves is tenable on these data sets. Application of the demand curves so-derived to publicly provided outputs requires an assumption that the demands for the outputs are the same regardless of production source.

2.) For recreation (including recreational uses of water and fish and wildlife), both travel cost and bidding game techniques use fixed price surrogates to trace out the demand curve, so no simultaneous supply estimation is necessary to identify demand.

3.) For non-commercial, non-recreational uses of water and fish and wildlife, valuation procedures generally center on bidding games which, again, use hypothetical price surrogates rather than supply shifts to identify demand.